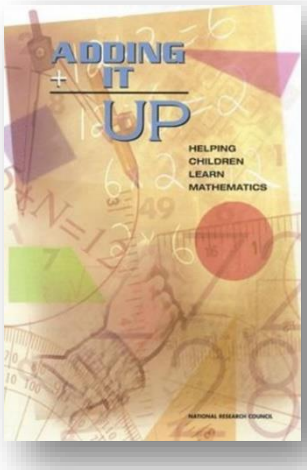
# The Components of Mathematical Proficiency

## Strategic Competence




*Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation in the literature of mathematics education and cognitive science, and mathematical problem solving, in particular, has been studied extensively. Although in school, students are often presented with clearly specified problems to solve, outside of school they encounter situations in which part of the difficulty is to figure out exactly what the problem is. Then they need to formulate the problem so that they can use mathematics to solve it.

Consequently, they are likely to need experience and practice in problem formulating as well as in problem solving. They should know a variety of solution strategies as well as which strategies might be useful for solving a specific problem. For example, sixth graders might be asked to pose a problem on the topic of the school cafeteria. Some might ask whether the lunches are too expensive or what the most and least favourite lunches are. Others might ask how many trays are used or how many cartons of milk are sold. Still others might ask how the layout of the cafeteria might be improved. With a formulated problem in hand, the student's first step in solving it is to represent it mathematically in some fashion, whether numerically, symbolically, verbally, or graphically. Fifth graders solving problems about getting from home to school might describe verbally the route they take or draw a scale map of the neighbourhood.

Representing a problem situation requires, first, that the student build a mental image of its essential components. Becoming strategically competent involves an avoidance of "number grabbing" methods (in which the student selects numbers and prepares to perform arithmetic operations on them) in favour of methods that generate problem models (in which the student constructs a mental model of the variables and relations described in the problem). To represent a problem accurately, students must first understand the situation, including its key features. They then need to generate a mathematical representation of the problem that captures the core mathematical elements and ignores the irrelevant features. This step may be facilitated by making a drawing, writing an equation, or creating some other tangible representation…

Extracts from: National Research Council (2001) Adding it up: *Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Not only do students need to be able to build representations of individual situations, but they also need to see that some representations share common mathematical structures. Novice problem solvers are inclined to notice similarities in surface features of problems, such as the characters or scenarios described in the problem. More expert problem solvers focus more on the structural relationships within problems, relationships that provide the clues for how problems might be solved… Flexibility develops through the broadening of knowledge required for solving nonroutine problems rather than just routine problems.

Routine problems are problems that the learner knows how to solve based on past experience. When confronted with a routine problem, the learner knows a correct solution method and is able to apply it. Routine problems require reproductive thinking; the learner needs only to reproduce and apply a known solution procedure… In contrast, nonroutine problems are problems for which the learner does not immediately know a usable solution method. Nonroutine problems require productive thinking because the learner needs to invent a way to understand and solve the problem.

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