Title: Consecutive Numbers Task

Purpose: To develop students' ability to generate and evaluate mathematical arguments

> **'Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers'**
>
> **Investigate this statement.**
> **What conclusions can be drawn?**
> **Justify your reasoning.**

This task can be used to develops students' understanding of connections between the Number and Algebra and Functions strands and can be used to strengthen students' understanding of generating and evaluating mathematical arguments. It is suggested that this should be a focus of the plenary. Please note, this task is not an example of a Classroom-Based Assessment.

Students should be encouraged to explain and justify their reasoning in relation to the generalisation or mathematical argument they generate. Reasoning is often done verbally, and accurate use of mathematical language should be developed. Students should also be encouraged to use the *Problem-Solving Toolkit* from CPD day 2018-19 to identify and note the tools and skills they are using and learning about. Where appropriate, students should be alerted to the difference between justification and formal proof.

The task is aligned to many of the Unifying strand's learning outcomes from the Junior Cycle Mathematics specification. For example, the teacher may choose to focus on:

- U4 represent a mathematical situation in a variety of different ways, including numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
- U10 evaluate different possible solutions to a problem, including evaluating the reasonableness of the solutions, and exploring possible improvements and/or limitations of the solutions (if any)
- U11 generate and evaluate mathematical statements or conjectures based on specific instances

The task is aligned to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

- N1 investigate the representation of numbers and arithmetic operations so that they can:

  a) represent the operations of addition, subtraction, multiplication, and division in $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ using models including the number line, decomposition, and accumulating groups of equal size
  c) explore numbers written as a^b

- N4 analyse numerical patterns in different ways, including making out tables and graphs and continue such patterns
- AF1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in numbers, spatial patterns and real-world phenomena involving change so that they can:
  a) represent these patterns and relationships in tables and graphs
  b) generate a generalised expression for linear **and quadratic** patterns in words and algebraic expressions and fluently convert between each representation
  c) categorise patterns as linear, non-linear, **quadratic, and exponential (doubling and tripling)** using their defining characteristics as they appear in the different representations
- AF2 investigate situations in which letters stand for quantities that are variable so that they can:
  a) generate and interpret expressions in which letters stand for numbers
  b) find the value of expressions given the value of the variables

It may be necessary to reduce the difficulty level of the task by selectively using effective questioning to guide students towards a generalisation or mathematical argument. It is recommended, however, that scaffolding be kept to a minimum to allow students to apply their knowledge in unfamiliar situations and create a need for the construction of new knowledge. If the task is scaffolded students may engage with it multiple times over the three years of Junior Cycle. Teachers can encourage student dialogue using effective questioning and active listening. Below are some examples of questions that reinforce a student's prior knowledge and/or challenge their misconceptions. These questions may also provide an opportunity for extension and enrichment for students who may have fully grasped the concepts in primary school. A Quick Reference Guide to the Links between the Primary and Post-Primary Curricula can be found here.

This task can be used with students at the beginning, during or at the end of their initial study of Natural Numbers. If the task is used at the beginning or during a unit of learning the task is formative as it provides an opportunity for the class teacher to assess the level of knowledge of their students regarding the identification, sorting, organising and categorisation of various patterns which arise, and adapt the focus of learning.

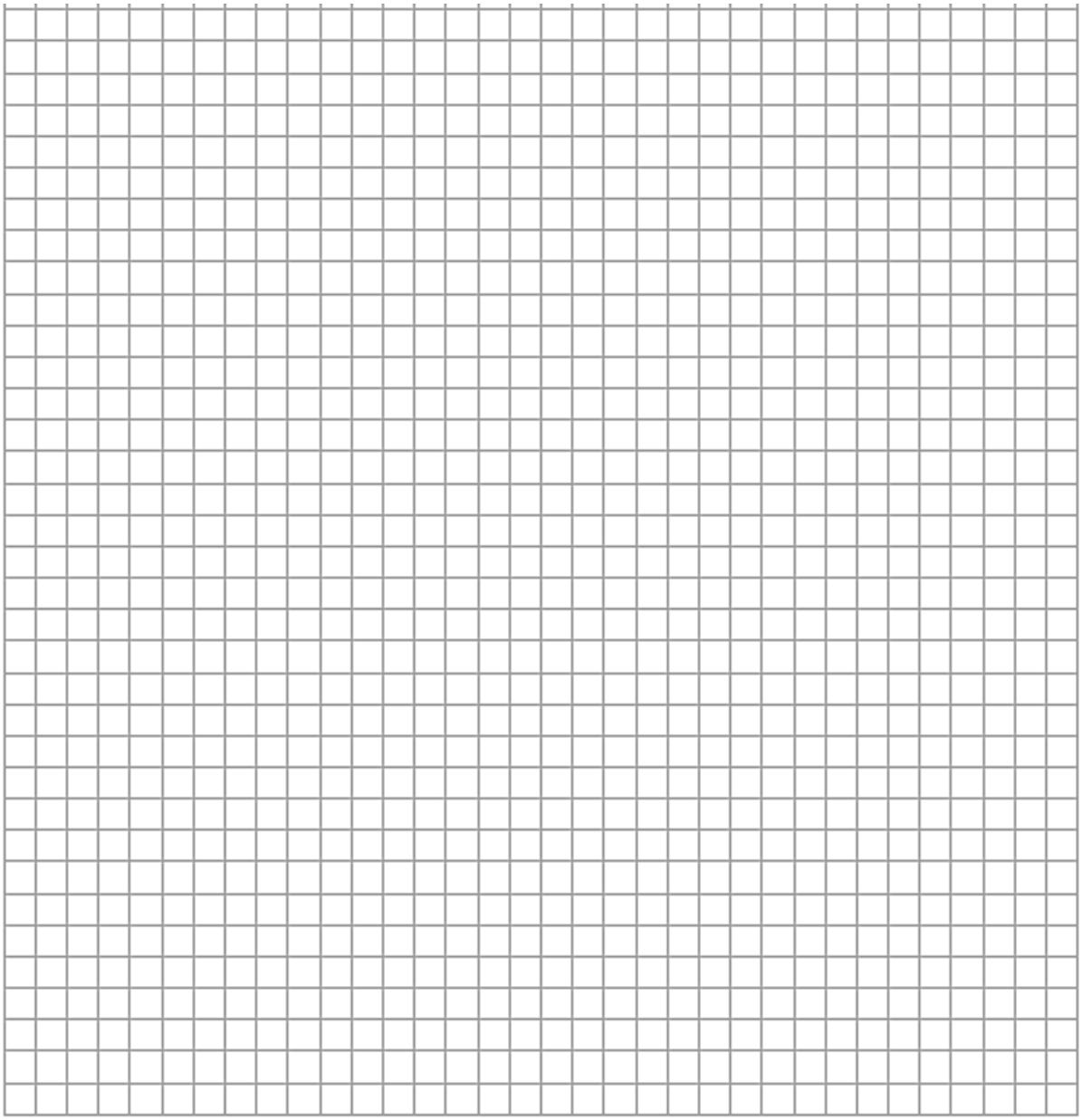
To promote students' mathematical thinking and discussion, and to generate rich classroom dialogue it is recommended that the task be undertaken in groups of between two and four students.

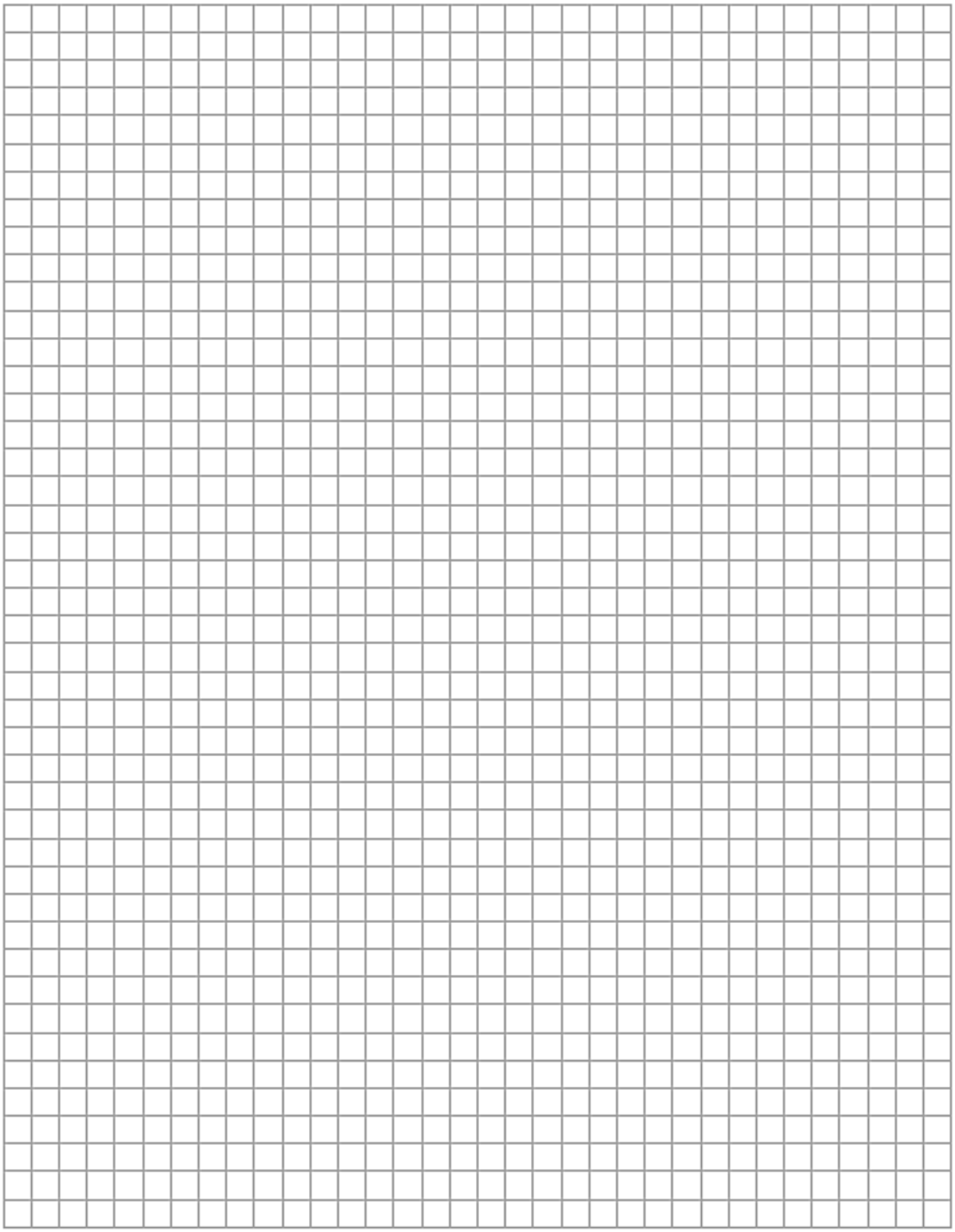
Suggested instructions for using this task:

- The teacher should introduce the task by formatively assessing the students current understanding of *Natural Numbers, Consecutive Numbers, Positive Numbers and Whole Numbers*. (It may be useful to check Learner Context using a Terminology Inventory Probe here)
- Each group should be given copy of the Consecutive Numbers Task.
- An initial opportunity to engage with the statement should be limited to *5 minutes*. Students should be encouraged to use examples to support their decisions.
- Teachers can encourage classroom dialogue through effective questioning and active listening.
- After consensus has been reached on the statement, the groups should be given a second opportunity to engage with the statement and encouraged to use the *Problem-Solving Toolkit* to identify the mathematical tools they have used or learned about.
- The teacher should move around the room informally gathering information (evidence) about student's knowledge and learning. Appropriate questioning often provides greater insight.
- Once the task has been completed, a plenary discussion is recommended. This should be informed by an information (evidence) gathering process and where appropriate questioning has the potential to enrich the class discussion and student learning.

‘Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers’

Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.





Sample Questions for Deeper Learning (This is not an exhaustive list)

'Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers' **Investigate this statement.** **What conclusions can be drawn?** **Justify your reasoning.**	What examples show that this is *Always, Sometimes, Never* true? Can you explain your conclusion? What would happen if you examined odd numbers only? What would happen if you examined the even numbers only?
1 + 2 = 3 2 + 3 = 5 3 + 4 = 7 4 + 5 = 9 …	What happens if you sum two consecutive numbers? Does this keep happening? What examples show this? What rule explains this best? Can you make a generalisation?
1 + 2 + 3 = 6 2 + 3 + 4 = 9 3 + 4 + 5 = 12 4 + 5 + 6 = 15…	What happens if you sum three consecutive numbers? Does this keep happening? What examples show this? What rule explains this best? Can you make a generalisation?
2 = 3 = 1 + 2 4 = 5 = 2 + 3 6= 1+2+3 7= 3+4 8= 9= 2+3+4	What would happen if you examined the numbers that cannot be made by summing consecutive numbers? Which natural numbers cannot be represented as a sum of consecutive numbers? Is there a pattern here? Can you make a generalisation?

Appendix I

Terminology Inventory Probe - Numbers

Natural numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Integers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Real numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Decimals

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Fractions

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Whole numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Consecutive numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Odd numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Even numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Square numbers

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Sum

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Product

- ❑ I have never heard of these.
- ❑ I have heard of these but I'm not sure what they are.
- ❑ I have some idea what these are.
- ❑ I know what these are and I can give an explanation.

Appendix II

Possible Extension Tasks

Below are some alternative extension tasks that allow for the links between Number and Algebra to be exposed.

- The difference of two consecutive numbers is an odd number.
- The sum of two consecutive triangular numbers is a square number.
- The sum of three consecutive triangular numbers is one more than three times the middle triangular number.
- The sum of three consecutive numbers is divisible by three.
- The sum of five consecutive numbers is divisible by five.
- The sum of four consecutive numbers is divisible by two.
- The sum of six consecutive numbers is divisible by three.
- One more than the sum of four consecutive numbers is even.
- The product of two consecutive numbers is divisible by two.
- The product of three consecutive numbers is divisible by six.
- One more than four times the product of two consecutive numbers is a perfect square.
- One more then the product of two numbers differing by two is a perfect square.
- One more then the product of four consecutive numbers is the square of one less than the product of the two middle numbers.
- The product of three consecutive numbers is the middle number less than a perfect cube.
- One more than eight times a triangular number is always a perfect square.